

# Nobel Lecture: LIGO and gravitational waves III\*

Kip S. Thorne

California Institute of Technology, Pasadena, California 91125, USA

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## CONTENTS

I. Introduction and Overview	1
II. Some Early Personal History: 1962–1976 <sup>1</sup>	1
III. Sources of Gravitational Waves	3
IV. Information Carried by Gravitational Waves, and Computation of Gravitational Waveforms	5
A. Observables from a compact binary’s inspiral waves	5
B. Post-Newtonian approximation for computing inspiral waveforms	5
C. Numerical relativity for computing merger waveforms	6
D. Geometrodynamics in BBH mergers	9
V. Theorists’ Contributions to Understanding and Controlling Noise in the LIGO Interferometers	11
A. Scattered-light noise	11
B. Gravitational noise	11
C. Thermal noise	11
D. Quantum noise and the standard quantum limit for a gravitational interferometer	11
E. Quantum fluctuations, quantum nondemolition, and squeezed vacuum	13
VI. The Future: Four Gravitational Frequency Bands	14
A. LISA: The Laser Interferometer Space Antenna	14
B. PTAs: Pulsar timing arrays	15
C. CMB polarization	15
VII. The Future: Probing the Universe with Gravitational Waves	16
A. Multi-messenger astronomy	16
B. Exploring black holes and geometrodynamics with gravitational waves	16
C. Exploring the first one second of our Universe’s life	17
VIII. Conclusion	18
Acknowledgments	18
References	18

## I. INTRODUCTION AND OVERVIEW

The first observation of gravitational waves, by LIGO on September 14, 2015, was the culmination of a near half century effort by ~1200 scientists and engineers of the LIGO/Virgo Collaboration. It was also the remarkable beginning of a whole new way to observe the universe: gravitational astronomy.

The Nobel Prize for “decisive contributions” to this triumph was awarded to only three members of the Collaboration: Rainer Weiss, Barry Barish, and me. But, in fact, it is the entire collaboration that deserves the primary credit. For this reason,

\*The 2017 Nobel Prize for Physics was shared by Rainer Weiss, Barry C. Barish, and Kip S. Thorne. These papers are the text of the address given in conjunction with the award.

in accepting the Nobel Prize, I regard myself as an icon for the Collaboration.

Because this was a collaborative achievement, Rai, Barry and I have chosen to present a single, unified Nobel Lecture, in three parts. Although my third part may be somewhat comprehensible without the other two, readers can only fully understand our Collaboration’s achievement, how it came to be, and where it is leading, by reading all three parts. Our three-part written lecture is a detailed expansion of the lecture we actually delivered in Stockholm on December 8, 2017.

In Part 1 of this written lecture, Rai describes Einstein’s prediction of gravitational waves, and the experimental effort, from the 1960s to 1994, that underpins our discovery of gravitational waves. In Part 2, Barry describes the experimental effort from 1994 up to the present (including our first observation of the waves), and describes what we may expect as the current LIGO detectors reach their design sensitivity in about 2020 and then are improved beyond that. In my Part 3, I describe the role of theorists and theory in LIGO’s success, and where I expect gravitational-wave astronomy, in four different frequency bands, to take us over the next several decades. But first, I will make some personal remarks about the early history of our joint experimental/theoretical quest to open the first gravitational-wave window onto the universe.

## II. SOME EARLY PERSONAL HISTORY: 1962–1976<sup>1</sup>

I fell in love with relativity when I was a teenage boy growing up in Logan, Utah, so it was inevitable that I would go to Princeton University for graduate school and study under the great guru of relativity, John Archibald Wheeler. I arrived at Princeton in autumn 1962, completed my Ph.D. in spring 1965, and stayed on for one postdoctoral year. At Princeton, Wheeler inspired me about black holes, neutron stars, and gravitational waves: relativistic concepts for which there was not yet any observational evidence; and Robert Dicke inspired and educated me about experimental physics, and especially experiments to test Einstein’s relativity theory.

In the summer of 1963 I attended an eight-week summer school on general relativity at the *École d’Été de Physique Théorique* in Les Houches, France. There I was exposed to the elegant mathematical theory of gravitational waves in lectures by Ray Sachs, and to gravitational-wave experiment in lectures by Joe Weber. Those lectures and Wheeler’s influence, together with conversations I had with Weber while hiking in the surrounding Alpine mountains, got me hooked

<sup>1</sup>For greater personal detail that feeds into this lecture, see my Nobel Biography.

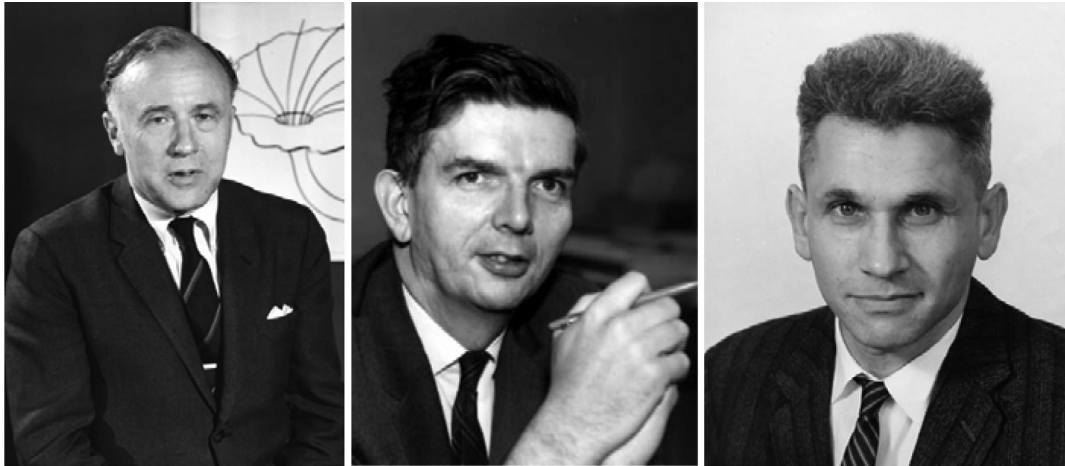


FIG. 1. John Wheeler, Robert Dicke, and Joseph Weber. Credit: Wheeler: AIP Emilio Segrè Visual Archives, Wheeler Collection. Dicke: Department of Physics, Princeton University. Weber: AIP Emilio Segrè Visual Archives.

on gravitational waves as a potential research direction. So it was inevitable that in 1966, when I moved from Princeton to Caltech and began building a research group of six graduate students and three postdocs, I focused my group on black holes, neutron stars, and gravitational waves.

My group's gravitational-wave research initially was quite theoretical. We focused on gravitational radiation reaction (whether and how gravitational waves kick back at their source, like a gun kicks back when firing a bullet). More importantly, we developed new ways of computing, accurately, the details of the gravitational waves emitted by astrophysical sources such as spinning, deformed neutron stars, pulsating neutron stars, and pulsating black holes. Most importantly (relying not only on our own group's work but also on the work of colleagues elsewhere) we began to develop a *vision for the future of gravitational-wave astronomy*: What would be the frequency bands in which observations could be made, what might be the strongest sources of gravitational waves in each band, and what information might be extractable from the sources' waves. We described this evolving vision in a series of review articles, beginning with one by my student Bill Press and me in 1972 (Press and Thorne, 1972) and continuing onward every few years until 2001 (Cutler and Thorne, 2002) when, with colleagues, I wrote the scientific case for the Advanced-LIGO gravitational-wave interferometers (Thorne *et al.*, 2001).

Particularly important to our evolving vision was the extreme difference between the electromagnetic waves with which astronomers then studied the universe, and the expected astrophysical gravitational waves:

- Electromagnetic waves (light, radio waves, X-rays, gamma rays, ...) are oscillating electric and magnetic fields that propagate through spacetime. Gravitational waves, by contrast, are oscillations of the “fabric” or shape of spacetime itself. The physical character of the waves could not be more different!
- Electromagnetic waves from astrophysical sources are almost always incoherent superpositions of emission produced by individual charged particles, atoms, or molecules. Astrophysical gravitational waves, by contrast, are emitted coherently by the bulk motion of mass or energy. Again, the two could not be more different.

- Astrophysical electromagnetic waves are all too easily absorbed and scattered by matter between their source and Earth. Gravitational waves are never significantly absorbed or scattered by matter, even when emitted in the earliest moments of the Universe's life.

These huge differences implied, it seemed to me, that

- Many gravitational-wave sources will not be seen electromagnetically.
- Just as each new electromagnetic frequency band (or “window”) when opened—radio waves, X-rays, gamma rays, ...—had brought great surprises due to the difference between that band and others, so gravitational waves with their far greater difference from electromagnetic waves are likely to bring even greater surprises.
- Indeed, gravitational astronomy has the potential to revolutionize our understanding of the universe.

In 1972, while Bill Press and I were writing our first vision paper, Rai Weiss at MIT was writing one of the most remarkable and prescient papers I have ever read (Weiss, 1972). It proposed an L-shaped laser interferometer gravitational-wave detector (*gravitational interferometer*) with free swinging mirrors, whose oscillating separations would be measured via laser interferometry. The bare-bones idea for such a device had been proposed earlier and independently by Michael Gertsenshtein and Vladislav Pustovoit in Moscow (Gertsenshtein and Pustovoit, 1963), but Weiss and only Weiss identified the most serious noise sources that it would have to face, described ways to deal with each one, and estimated the resulting sensitivity to gravitational waves. Comparing with estimated wave strengths from astrophysical sources, Rai concluded that such an interferometer with kilometer-scale arm lengths had a real possibility to discover gravitational waves. (This is why I regard Rai as the primary inventor of gravitational interferometers.)

Rai, being Rai, did not publish his remarkable paper in a normal physics journal. He thought one should not publish until after building the interferometer and finding gravitational waves, so instead he put his paper in an internal MIT report series, but provided copies to colleagues.

I heard about Rai's concept for this gravitational interferometer soon after he wrote his paper and while John Wheeler, Charles Misner, and I were putting the finishing touches on

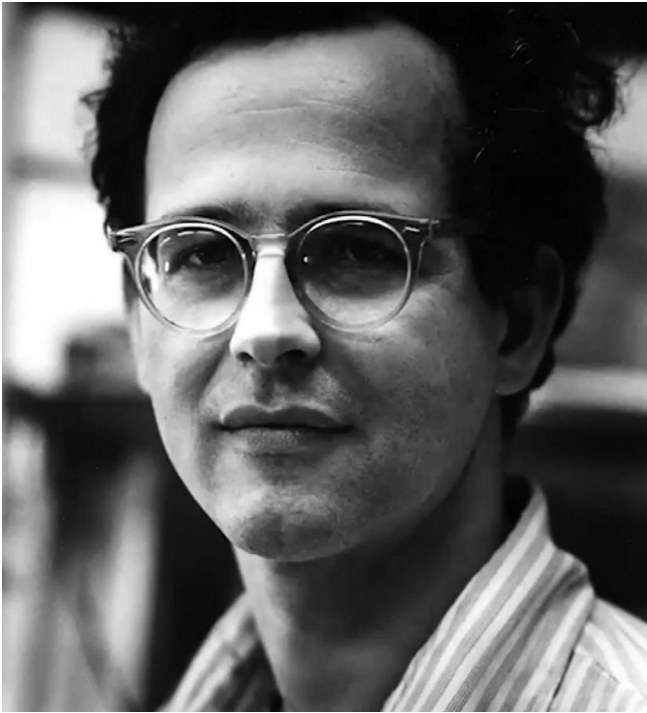


FIG. 2. Rainer Weiss ca. 1970. Credit: Rainer Weiss.

our textbook *Gravitation* (Misner, Thorne, and Wheeler, 1973) and preparing to send it to our publisher. I had not yet studied Rai’s paper nor discussed his concept with him, but it seemed very unlikely to me that his concept would ever succeed. After all, it required measuring motions of mirrors a trillion times smaller ( $10^{-12}$ ) than the wavelength of the light used to measure the motions—that is, in technical language, splitting a fringe to one part in  $10^{12}$ . This seemed ridiculous, so I inserted a few words about Rai’s gravitational interferometer into our textbook, and labeled it “not promising”.

Over the subsequent three years I learned more about Rai’s concept, I discussed it in depth with him (most memorably in 1975, in an all-night-long conversation in a hotel room in Washington, D.C.), and I discussed it with others. And I became a convert. I came to understand that Rai’s gravitational interferometer had a real possibility of discovering gravitational waves from astrophysical sources.

I was also convinced that, if gravitational waves could be observed, they would likely revolutionize our understanding of the universe; so I made the decision that I and my theoretical-physics research group should do everything possible to help Rai and his experimental colleagues discover gravitational waves. My major first step was to persuade Caltech to create an experimental gravitational-wave research group working in parallel with Rai’s group at MIT.

Rai sketches the rest of this history, on the experimental side, in his Part I of our Nobel Lecture, and I recount some of it in my Nobel biography. I now sketch the theory side of the subsequent history.

### III. SOURCES OF GRAVITATIONAL WAVES

When Bill Press and I wrote our 1972 vision paper, our understanding of gravitational-wave sources was rather

muddled, but by 1978 the relativistic astrophysics community had converged on a much better understanding. The convergence was accelerated by a two week *Workshop on Sources of Gravitational Waves* convened by Larry Smarr in Seattle, Washington in July–August 1978. The participants included almost all of the world’s leading gravitational-wave theorists and experimenters, plus a number of graduate students and postdocs: Figure 3.

Some conclusions of the workshop were summarized in diagrams depicting the predicted gravitational-wave strain  $h$  as a function of frequency  $f$  for various conceivable sources (Epstein and Clark, 1979): three diagrams, one for short-duration (“burst”) waves, one for long-duration, periodic waves (primarily from pulsars and other spinning, deformed neutron stars), and one for stochastic waves (primarily, we thought then, superpositions of emission from many discrete sources). Most relevant to this lecture is the segment of the burst-wave diagram that covers LIGO’s frequency band: Figure 4.

The waves here depicted are from:

- *Supernovae* (SN), that is, the implosion of the core of a normal star to form a neutron star, releasing enormous gravitational energy that blows off the normal star’s outer layers.
- *Compact-binary destruction* (CBD), that is, the inspiral and merger of binaries consisting of two black holes, two neutron stars, or a black hole and a neutron star.

The supernova line in the figure was an estimated upper limit on the strengths of the waves from supernovae. More modern estimates predict waves much weaker. The box labeled CBD was the range in which the strongest compact-binary waves were expected.

Looking at this figure, we workshop participants concluded that the strongest gravitational-wave burst reaching Earth each year would have an amplitude of roughly  $h \sim 10^{-21}$ ; and I (mis) remember that in our enthusiasm for this goal, we had T-shirts made up with the logo on them “ $10^{-21}$  or bust”. However, colleagues with better memories than mine assure me we only discussed such T-shirts; the T-shirts were never actually created.

The first wave burst that LIGO finally detected, in 2015, was at the location of the red star, which I have added to this figure, and was from CBD: the inspiral and merger of two black holes (a “binary black hole” or BBH). Its amplitude was precisely  $10^{-21}$  and its frequency was about 200 Hz—a bit stronger strain  $h$  and lower frequency than our 1978 estimates. This agreement of prediction and observation is partially luck. Our level of knowledge in 1978 was much lower than it suggests.

By 1984, when Weiss, Drever and I were co-founding the LIGO Project, I thought it likely that the strongest waves LIGO would detect would come from the merger of binary black holes (as did happen). My reasoning was simple:

- The amplitude of a compact binary’s gravitational-wave strain  $h$  is proportional to the binary’s mass (if its two objects have roughly the same mass).
- Therefore the distance to which LIGO can see it is also proportional to its mass (so long as the waves are in LIGO’s frequency band), which means for binary masses between a few suns and a few hundred suns, i.e. “stellar-mass” compact binaries.



FIG. 3. Participants in the 1978 workshop on gravitational waves. Credit: Larry Smarr.

- Correspondingly, the volume within which LIGO can see such binaries is proportional to the cube of the binary's mass.
- The masses of then-known stellar-mass black holes were as much as 10 times greater than those of neutron stars, so the volume searched would be 1000 times greater than for neutron stars.
- It seemed likely to me that this factor 1000 would outweigh the (very poorly understood) lower number of BBH in the universe than binary neutron stars, BNS.

Although this was just a guess, in planning for LIGO it led us to lay heavy emphasis on binary black holes, as well as on the much better understood binary neutron stars.

By 1989 when, under the leadership of Rochus (Robbie) Vogt, we wrote our construction proposal for LIGO (Vogt *et al.*, 1989) and submitted it to NSF, gravitational waves from compact binaries were central to our arguments for how sensitive our gravitational interferometers would have to be.

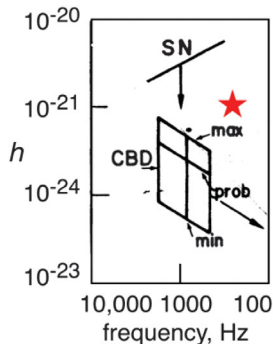


FIG. 4. Segment of 1978 burst-source diagram.

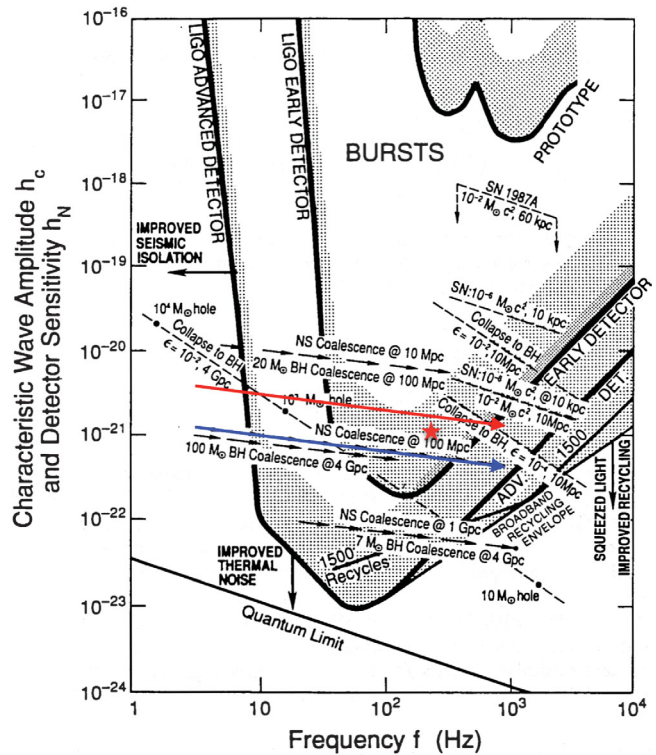


FIG. 5. Figure A-4a from the 1989 construction proposal for LIGO, showing estimates of noise curves (solid) for Initial and Advanced-LIGO interferometers, and the estimated strengths of waves from various sources. The tops of the stippled regions are the strength that a signal would need for confident detection with Gaussian noise and optimal signal processing. The quantum limit is for 1000 kg mirrors.

The estimated event rates and strengths were so crucial to the scientific case for LIGO that we thought it essential to rely on rate estimates from astrophysicists who had no direct association with our project. For binary neutron stars (BNS), those estimates (Clark, Van den Heuvel, and Sutantyo, 1979) (based on the statistics of observed binary pulsars in our own Milky Way galaxy) placed the nearest BNS merger each year somewhere in the range of 60 to 200 Mpc, with a most likely distance of 100 Mpc (320 million light years), and a signal strength as shown by the blue, arrowed line in Figure 5. (In 2017, when the first BNS was observed, its distance was about 40 Mpc—somewhat closer than expected—and its strength was as shown by the red, arrowed line in the figure.) For BBH merger rates, the uncertainties in 1989 remained so great that we did not quote estimates. (The first BBH seen, in 2015, was as shown by the red star.)

In the 1990s and 2000s, astrophysicists made more reliable estimates of BBH and BNS waves, with less than a factor 2 change in the BNS distances, and with the distance for the nearest BBH getting narrowed down to a factor  $\sim 10$  uncertainty ( $\sim 1000$  uncertainty in the rate of bursts) (LIGO/Virgo, 2010).

#### IV. INFORMATION CARRIED BY GRAVITATIONAL WAVES, AND COMPUTATION OF GRAVITATIONAL WAVEFORMS

##### A. Observables from a compact binary's inspiral waves

In 1986 Bernard Schutz (Schutz, 1986) (one of the leaders of the British-German gravitational-wave effort) identified the *observables* (parameters) that can be extracted from the early inspiral phase of a compact binary's gravitational waves. From the gravitational-wave strain  $h$  as a function of time  $t$ ,  $h(t)$ ,



FIG. 6. Bernard Schutz. Credit: Bernard F. Schutz.

measured at several locations on Earth, one can infer, he deduced:

- The direction to the binary.
- The inclination of its orbit to the line of sight.
- The direction the two objects move around their orbit.
- The *chirp mass*,  $M_c = (M_1 M_2)^{3/5} / (M_1 + M_2)^{1/5}$  (where  $M_1$  and  $M_2$  are the individual masses).
- The distance  $r$  from Earth to the binary (more precisely, in technical language, the binary's *luminosity distance*).

It is remarkable that gravitational astronomy gives us the binary's distance  $r$  but not its redshift  $z$  (fractional change in wavelengths due to motion away from Earth), whereas electromagnetic astronomy, looking at the same binary, can directly measure its redshift but not its distance. In this sense, gravitational and electromagnetic observations are complementary, not duplicative.

The relationship between distance and redshift,  $r(z)$ , is crucial observational data for cosmology; for example, if the binary is not too far away,  $r(z)$  determines the Hubble expansion rate of the universe today. Therefore, as Schutz emphasized, for binary neutron stars it should be possible to observe both the binary's gravitational waves (distance) and its electromagnetic waves (redshift) and thereby explore cosmology. That is precisely what happened in 2017 with LIGO's discovery of its first BNS, GW170817; see Barish's Part II of this lecture.

[In 1986, having identified the gravitational-wave observables for compact binaries, Schutz then started laying foundations for the analysis of data from gravitational interferometers (Schutz, 1989). He became the intellectual leader of this effort in the early years, before I or anyone else in LIGO began thinking seriously about data analysis. For some discussion of LIGO data analysis, see Weiss's and Barish's Parts I and II of this lecture.]

As a compact binary spirals inward due to radiation reaction, the strength of the mutual gravity of its two bodies grows larger, their speeds grow higher, and correspondingly, relativistic effects (deviations from Newton's laws of gravity) become stronger. This presents a *problem* (the need to compute relativistic corrections to the binary's waveforms), and an *opportunity* (the possibility that those corrections, when observed, will bring us additional information about the binary and can be used to test general relativity in new ways).

##### B. Post-Newtonian approximation for computing inspiral waveforms

The relativistic corrections are computed, in practice, using the *post-Newtonian approximation* to general relativity: a power-series expansion in powers of the bodies' orbital velocities  $v$  and their Newtonian gravitational potential  $\Phi \sim v^2$ . Motivated by the astronomical importance of these waveform corrections, several efforts were mounted to compute them beginning in the 1970s, and then the efforts accelerated in the 1980s, 1990s, and 2000s. I estimate that many more than 100 person years of intense work were put into this effort. The leading contributors included, among others, Luc Blanchet, Thibault Damour, Bala Iyer, and Clifford Will; and by now the computations have

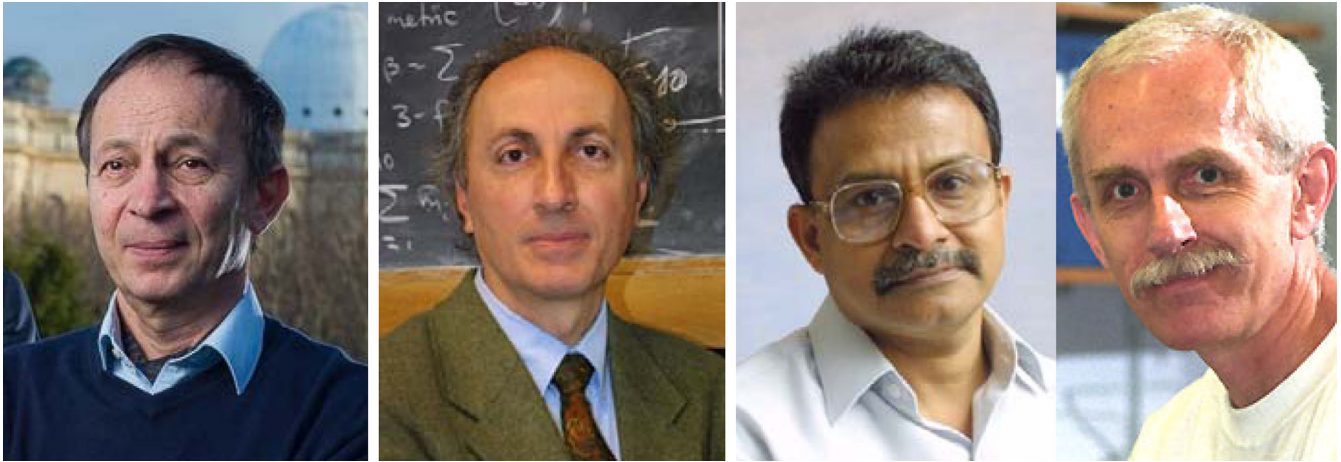


FIG. 7. Luc Blanchet, Thibault Damour, Bala Iyer, and Clifford Will. Credits: Blanchet: Luc Blanchet. Damour: Thibault Damour. Iyer: Bala Iyer. Will: Clifford M. Will.

been carried up to order  $v^7$  beyond Newton's theory of gravity (Blanchet, 2014). As expected, at each higher order in the computation, there are new observables that can be extracted from the observed waves. These include, most importantly, the individual masses  $M_1$  and  $M_2$  of the binary's two bodies, and their vectorial spin angular momenta; and, if the binary's orbit is not circular, then its evolving ellipticity and elliptical orientation, and relativistic deviations from elliptical motion. And at each order, there are new opportunities to test, observationally, Einstein's general relativity theory—tests that are now being carried out with LIGO's observational data (Cutler *et al.*, 1993; LIGO/Virgo, 2016).

### C. Numerical relativity for computing merger waveforms

When the relative velocity of the binary's two bodies approaches  $1/3$  the speed of light and the bodies near collision, the post-Newtonian approximation breaks down. This, again, presents a *problem* (how to compute the waveforms) and an *opportunity* (new information carried by the waveforms).

The only reliable way to compute the waveforms in this collision epoch is by numerical simulations: solving Einstein's general relativistic field equations on a computer—*numerical relativity*. For this reason, in the 1980s I began urging my numerical relativity colleagues to push forward vigorously on such simulations.

Simulating BBHs was especially important, for several reasons:

- For neutron stars, with their small masses (about 1.4 suns each), the waves from the collision epoch are at such high frequencies that they will be difficult for LIGO to detect and monitor; almost all of the signal strength and extractable information will come from lower frequencies, where the post-Newtonian approximation is accurate.
- For black holes, by contrast, the collision epoch can produce waves at frequencies where LIGO is most sensitive. (That is precisely what happened with LIGO's

first observed wave burst, GW150914; almost all of its signal strength came from the collision epoch, which could be analyzed only via numerical relativity.)

- The waveforms from BBH collision and merger carry detailed information about *geometrodynamics*: the non-linear dynamics of curved spacetime—about which we knew very little in the 1980s and 90s.

In the late 1950s and early 1960s, John Wheeler identified geometrodynamics as tremendously important. It is the arena where Einstein's general relativity should be most rich, and deviations from Newton's laws of gravity should be the greatest. Black-hole collisions, Wheeler argued, would be an ideal venue for studying geometrodynamics. Recognizing the near impossibility of exploring geometrodynamics analytically, with pencil and paper, Wheeler encouraged his students and colleagues to explore it via computer simulations.

With this motivation, Wheeler's students and colleagues began laying foundations for BBH simulations: In 1959–1961, Charles Misner, Richard Arnowitt and Stanley Deser (Arnowitt, Deser, and Misner, 1962, and references therein) brought the mathematics of Einstein's equations into a form nearly ideal for numerical relativity, and Misner analytically solved the *initial-value* or *constraint* part of these equations to obtain a mathematical description of two black holes near each other and momentarily at rest (Misner, 1960). Then in 1963, Susan Hahn and Richard Lindquist (Hahn and Lindquist, 1964) solved the full Einstein equations numerically, on an IBM 7090 computer, and thereby watched the two black holes fall head-on toward each other and begin to distort each other. Sadly, Hahn and Lindquist could not compute long enough to see the holes' collision and merger, nor the gravitational waves that were emitted.

These calculations were picked up in the late 1960s, with some change in the detailed formulation, by Bryce DeWitt and DeWitt's student Larry Smarr, and were brought to fruition by Smarr and his student Kenneth Eppley in 1978 (Smarr, 1979, and references therein). In these simulations the two holes collided head on and merged to form a single, highly distorted black hole that vibrated a few times (rang like a damped bell), emitting a burst of gravitational waves, and then settled down



FIG. 8. John Wheeler lecturing about geometrodynamics and related issues at Willy Fowler's 60th birthday conference in August 1971, in Cambridge England. Fowler is the Nobel Laureate with the shiny bald head in the front row. Credit: Kip Thorne.

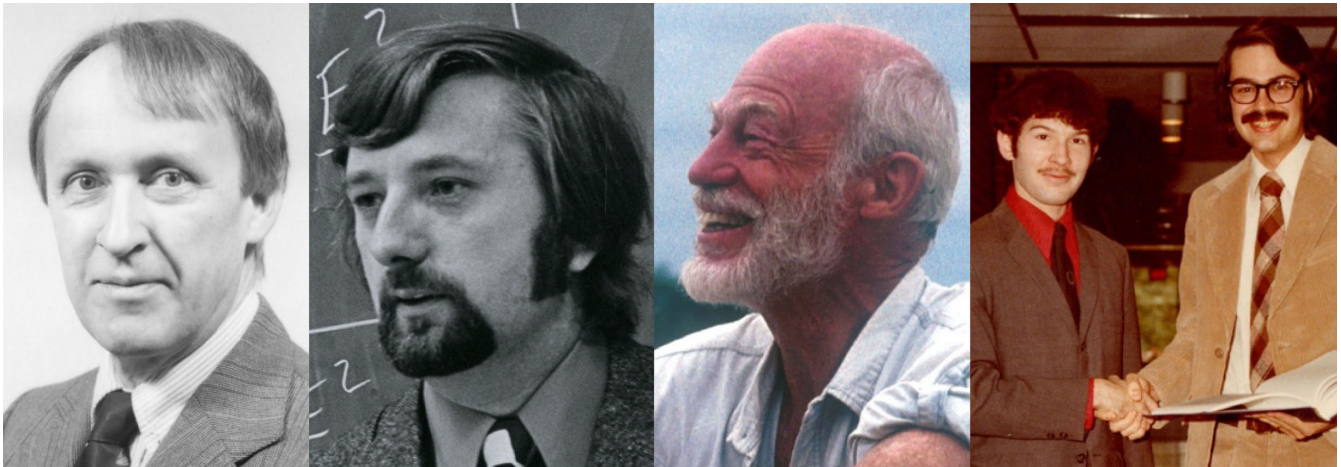


FIG. 9. Charles Misner, Richard Lindquist, Bryce DeWitt, Kenneth Eppley, and Larry Smarr. I have not been able to find a photo of Susan Hahn. Credits: Misner: Charles W. Misner. Lindquist: Wesleyan University Library, Special Collections & Archives. DeWitt: Kip Thorne. Eppley & Smarr: Larry Smarr.

into a quiescent state. Here we had, at last, our first example of geometrodynamics.

But head-on collisions should occur rarely, if ever, in Nature. When two black holes or stars orbit each other, gravitational radiation reaction drives their orbit into a circular form rather quickly, so BBH collisions and mergers should almost always occur in circular, inspiraling orbits. The big challenge for the 1980s and 1990s, therefore, was to simulate BBHs with shrinking, circular orbits.

This was so difficult that by 1992 only modest progress had been made. To accelerate the progress, Richard Isaacson (the NSF program director who had nurtured the LIGO experimental effort with great skill, see Weiss's Part I of this lecture) urged all the world's numerical relativity groups to collaborate on this problem, at least loosely. Richard Matzner of the University of Texas at Austin led this *Binary Black Hole Grand Challenge Alliance*, and I chaired its advisory committee. To generate collegiality and speed things up, in 1995 I bet many of the Alliance's members that

LIGO would observe gravitational waves from BBH mergers before numerical relativists could simulate the mergers; see Figure 10. I fervently hoped to lose, since the simulations would be crucial to extracting the information carried by the observed waves.

By early 2002, the Alliance had made much progress, but was still unable to simulate a full orbit of two black holes around each other. The computer codes would crash before an orbit was complete, and I was worried I might win the bet.

Alarmed, I left day to day involvement in the LIGO project and focused on helping push numerical relativity forward. Together with Lee Lindblom, I created a numerical relativity research group at Caltech, as an extension of the group I respected most: that of Saul Teukolsky at Cornell. With the help of private funding from the Sherman Fairchild Foundation, we grew our joint Cornell/Caltech *Program to Simulate eXtreme Spacetimes* (SXS) to the size we thought was needed for success: about 30 researchers.


*Kip Thorne hereby wagers that LIGO will discover convincing gravitational waves from black hole coalescence before the numerical relativity community has a code capable of computing merger waveforms, to 10 per cent accuracy, as determined by internal computational consistency, for coalescences with random spin directions and magnitudes and random mass ratios in the range 1:1 to 10:1. The signatories below wager that Kip is wrong.*

*The loser(s) will supply a bottle or bottles of wine, value not less than \$100, to be consumed by the winner(s) and loser(s) together.*

*Agreed to this 17th day of July, 1995 in Austin, Texas by:*

*Kip S. Thorne*

Richard Matzner  
Wai-Mo Suen  
Ed Seidel  
Mark Scheel  
Lawrence E. Kidder  
Gregory B. Cook  
Luciano Rezzolla  
Mark Miller (♫)  
Larry Shepley  
Shyamal Mitra  
Manoj Maharaj  
Daniel Holz  
Pablo Laguna  
Roberto Gomez



Jörg Frauendiener  
Dierdre Shoemaker  
Bernd Brügmann  
Béla Szilágyi  
Nigel Bishop  
Sascha Husa  
Jeff Winicour  
Mijan Huq  
Luis Lehner  
Robert Marsa  
Scott Klasky  
Marcus Berg  
Juan F. Lara  
Ethan Honda

*Richard Matzner*

FIG. 10. My bet with Richard Matzner (photo) and members of his Binary Black Hole Grand Challenge Alliance. Credit: Matzner: Richard Matzner.



FIG. 11. Franz Pretorius, Manuela Campanelli, Joan Centrella, and Saul Teukolsky. Credits: Pretorius: New York Academy of Sciences. Campanelli: A. Sue Weisler/RIT. Centrella: Dwight Allen. Teukolsky: Saul A. Teukolsky.

The SXS program's first great triumph arose not, however, from the collaborative work of the SXS team. Rather it was a single-handed triumph by Franz Pretorius, an SXS postdoc. In June 2005, Franz cobbled together a set of computational techniques and tools into a single computer code that successfully simulated the orbital inspiral, collision, and merger of a BBH, one whose black holes were identical and not spinning (Pretorius, 2005). Six months later, two other small research groups achieved the same thing, using rather different techniques and tools: a group led by Joan Centrella at NASA's Goddard Spaceflight Center, and another led by Manuela Campanelli at the University of Texas at Brownsville (Baker *et al.*, 2006; Campanelli *et al.*, 2006). I heaved a sigh of relief; perhaps I would actually lose my bet!

But we were still a long way from meeting LIGO's needs: It was necessary to simulate BBHs whose two black holes have masses that differ by as much as a factor of 10, and spin at different rates and in different directions. And these simulations had to be carried out with a computer code that was highly stable and robust, and had a well calibrated accuracy that matched LIGO's needs. And it was necessary

to carry out a large suite of simulations that covered the full range of parameters to be expected for LIGO's observed sources—seven non-trivial parameters: the ratio of the holes' masses, and the three components of the vectorial spin of each black hole. We estimated that about a thousand simulations would be needed in preparation for LIGO's early BBH observations.

To achieve this goal, Teukolsky led the SXS team in constructing a code based on a formulation of Einstein's equations that is strongly hyperbolic and uses spectral methods—technical details that guarantee the code's accuracy will improve *exponentially fast* as the coordinate grid is refined. The resulting SXS code is called *SpEC* for *Spectral Einstein Code*.<sup>2</sup>

SpEC was far more difficult to write and perfect than the Pretorius, Centrella, and Campanelli codes, or codes created by several other numerical relativity groups (notably Bernd Brugman's group in Jena, Germany, and Pablo

<sup>2</sup><http://www.black-holes.org/SpEC.html>.



Laguna’s Georgia Tech code, which grew out of Matzner’s Texas effort). The other codes were perfected several years before SpEC and made major discoveries about geometrodynamics while SpEC was still being perfected. But SpEC did reach perfection a few years before LIGO’s first BBH observation and then was used to begin building the large catalog of BBH waveforms to underpin LIGO data analysis<sup>3</sup>; and now that we are in the LIGO observational era, only SpEC has the speed and accuracy to fully meet LIGO’s near-term needs (Hinderer *et al.*, 2014). And with great relief, I have conceded the bet to my numerical relativity colleagues.

Interfacing the output of the numerical relativity codes with LIGO data analysis was a major challenge. The interface was achieved by a quasi-analytic model of the BBH waveforms called the *Effective One Body* (EOB) Formalism, which was devised by Alessandra Buonanno and Thibault Damour (Buonanno and Damour, 1999); and also achieved by the quasi-analytic *Phenomenological Formalism*, devised by Parameswaran Ajith and colleagues (Ajith *et al.*, 2007). The numerical relativity waveforms were used to tune parameters in these formalisms, which then were used to underpin the LIGO data-analysis algorithms that discovered the BBH waves and did a first cut at extracting their information. The final extraction of information is most accurately done by direct comparison with the SpEC simulations.

#### D. Geometrodynamics in BBH mergers

Just as I did not play a role in LIGO’s experimental R&D, so also I did not play any role at all in formulating and perfecting the SXS computer code SpEC. My primary role in both cases was more that of a visionary. For SpEC a big part of that vision was inherited from Wheeler: Use SpEC simulations of BBHs to predict the geometrodynamical excitations of curved spacetime that are triggered when two black holes collide, and then use LIGO’s observations to test those predictions.

By 2011, SpEC was mature enough to start exploring geometrodynamics. To assist in those explorations, we developed several visualization tools.

The first was a *pseudo-embedding diagram* (Figure 12), developed by SXS researcher Harald Pfeiffer. In this diagram, Pfeiffer takes the BBH’s orbital “plane” (a two-dimensional warped surface), and visualizes its warpage (or, in physicists’ language, its curvature) by depicting it embedded in a hypothetical, flat three-dimensional space. The colors of the resulting warped surface depict the slowing of time: in the green regions, time flows at roughly the same rate as far away; in the red regions, the rate of flow of time is greatly slowed; the black regions (not often visible) are inside the black hole, where time flows downward. The silver arrows depict the motion of space.<sup>4</sup>

<sup>3</sup><https://www.black-holes.org/for-researchers/waveform-catalog>.

<sup>4</sup>In more technical language, the surface’s shape, color, and arrows depict the 2-geometry of the orbital “plane”, the lapse function, and the shift function.

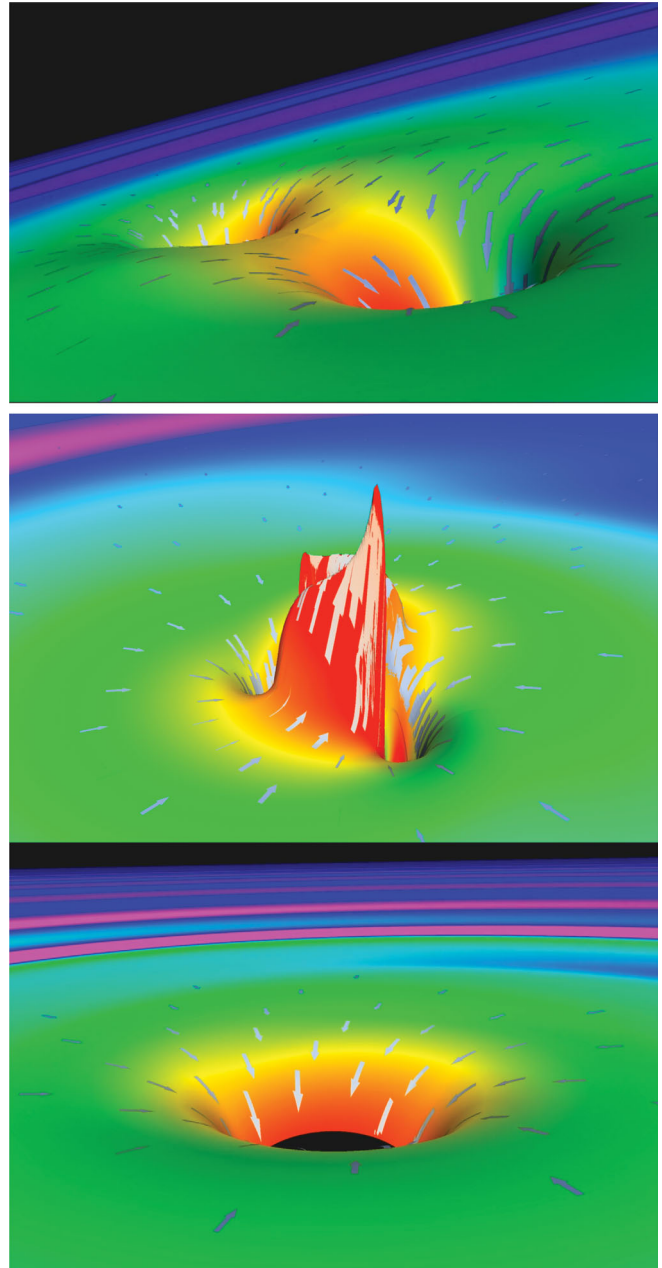


FIG. 12. Snapshots (pseudo-embedding diagrams) from a movie depicting the geometry of spacetime around the GW150914 binary black hole 60 ms before collision, at the moment of collision, and 12 ms after the collision. Credits: SXS Collaboration.

From a sequence of these diagrams (based on the output of an SXS simulation), Pfeiffer constructs a movie<sup>5</sup> of the BBH’s evolving spacetime geometry. Figure 12 shows three snapshots from the movie for a BBH whose parameters are those of the first gravitational-wave burst that LIGO observed, GW150914:

- The first snapshot shows the BBH 60 milliseconds before collision. The space around each black hole dips

<sup>5</sup><https://www.youtube.com/watch?v=YsZFRkzLGew>.

downward like the water surface in a whirlpool, and the color shifts from green to red (time slows) as one moves down the tube.

- The second snapshot shows the BBH at the moment of collision. The collision has created a veritable *storm* in the shape of spacetime: Space is writhing like the surface of the ocean in a weather storm, and the rate of flow of time is changing rapidly.
- The third snapshot shows the BBH after the storm has subsided. It has produced a quiescent, single, merged black hole; and far from the hole, a burst of gravitational waves (depicted only heuristically as water-wave-type ripples) flows out into the universe.

These pseudo-embedding diagrams and movie have serious limitations. They depict only the BBH's equatorial plane and not the third dimension of our universe's space. The gravitational waves are not well depicted because they are essentially three dimensional. And some remarkable phenomena are completely missed, for example, two *vortices* of twisting space (one with a clockwise twist, the other counter-clockwise) that emerge from each black hole, and also a set of

stretching and squeezing warped-spacetime structures called *tendices* (Owen *et al.*, 2011).

The SXS simulations reveal the rich geometrodynamics of the BBH's spacetime geometry, and of its vortices and tendices. And the beautiful agreements between LIGO's observed gravitational waveforms and those predicted by the SXS simulations (e.g. Figure 6 of Barish's Part II of this lecture) convince us that geometrodynamics storms really do have the forms that the simulations predict—i.e. that Einstein's general relativity equations predict.

If you and I were to watch two black holes spiral inward, collide and merge, with our own eyes or a camera, we would see something very different from the pseudo-embedding snapshots of Figure 12 and their underlying movie. Far behind the BBH would be a field of stars. The light from each star would follow several different paths to our eyes (Figure 13), some rather direct, others making loops around the black holes; so we would see several images of each star. (This is called *gravitational lensing*.) And as the holes orbit around each other, the images would move in a swirling pattern around the holes' two black shadows.

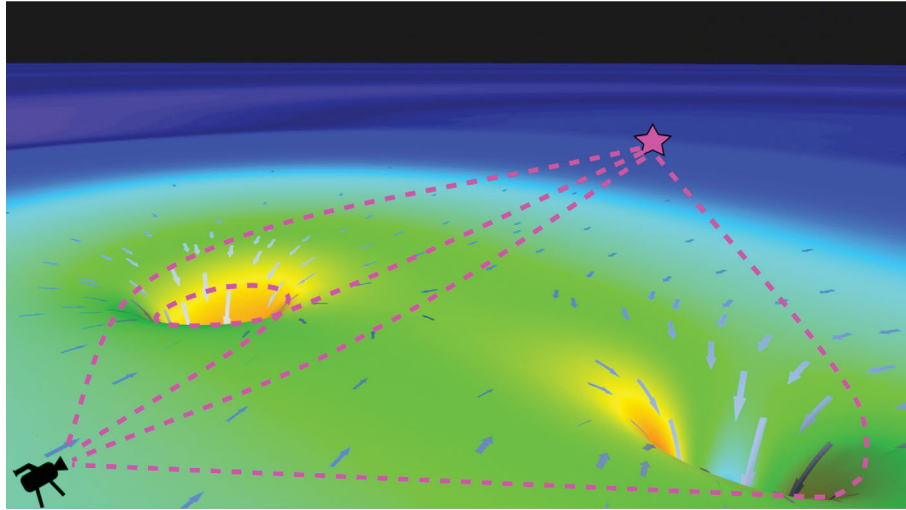


FIG. 13. Light rays from a star, through the warped spacetime of GW150914, to a camera. Adapted from the movie (see footnote 5) that underlies Fig. 12: SXS Collaboration.



FIG. 14. The BBH GW150914 as seen by eye, up close. Credit: SXS Collaboration.

Teukolsky's graduate students Andy Bohn, Francois Hébert, and Will Throwe produced a movie<sup>6</sup> (Bohn *et al.*, 2015) of these swirling stellar patterns from the SXS simulation of LIGO's first observed BBH, GW150914. Figure 14 is a snapshot from that movie.

Figures 12 and 14 and the geometrodynamical phenomena that I have described give a first taste of the exciting science that will be extracted from gravitational waves in the future. To that future science I will return below. But first I will dip back into the past, and describe briefly some contributions that theorists have made to the experimental side of LIGO.

## V. THEORISTS' CONTRIBUTIONS TO UNDERSTANDING AND CONTROLLING NOISE IN THE LIGO INTERFEROMETERS

A major aspect of the LIGO experiment is understanding and controlling a huge range of phenomena that produce noise which can hide gravitational-wave signals. Theorists have contributed to scoping out some of these phenomena. This has been highly enjoyable, and it has broadened the education of theory students. I will give several interesting examples:

### A. Scattered-light noise

In each arm of a LIGO interferometer the light beam bounces back and forth between mirrors. A tiny portion of the light scatters off one mirror, then scatters or reflects from the inner face of the vacuum tube that surrounds the beam, then travels to the other mirror, and there scatters back into the light beam (Figure 15, top). The tube face vibrates with an amplitude that is huge compared to the gravitational wave's influence, and those vibrations put a huge, oscillating phase shift onto the scattered light. That huge phase shift on a tiny fraction of the beam's light can produce a net phase shift in the light beam that is bigger than the influence of a gravitational wave.

This light-scattering noise can be controlled by placing baffles in the beam tube (dashed lines in Figure 15) to block the scattered light from reaching the far mirror. A bit of the scattered light, however, can still reach the far mirror by diffracting off the edges of the baffles.

Baffles and their diffraction of light are a standard issue in optical telescopes and other devices. But not standard, and unique to gravitational interferometers, is the danger that there might be *coherent* superposition of the oscillating phase shift for light that travels by different routes from one mirror to the other; such coherence could greatly increase the noise. In 1988 Rai Weiss recruited me and my theory students to look at this, determine how serious it is, and devise a way to mitigate it. Eanna Flanagan and I did so. To break the coherence, we gave the baffles deep saw teeth with random heights (Figure 15, bottom), and to minimize the noise further we chose the teeth pattern optimally and optimized the locations of the baffles in the beam tube (Flanagan and Thorne, 1995). A segment of one of our random-saw-toothed baffles is my contribution to the Nobel Museum in Stockholm.

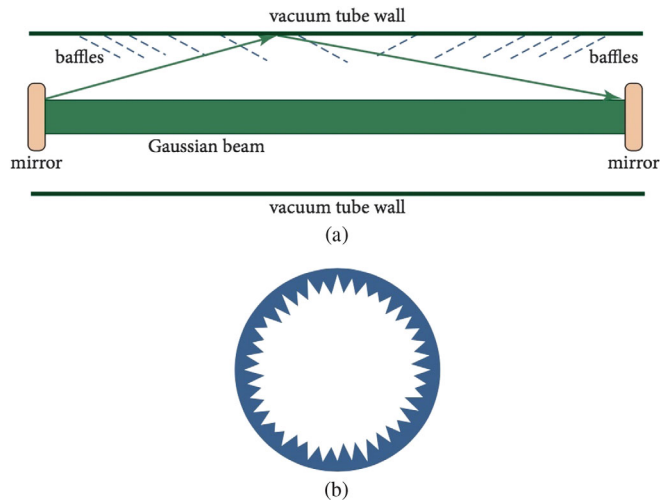


FIG. 15. Top: A bit of beam light scatters off LIGO mirror, then scatters off vacuum tube wall, then travels to far mirror, and then scatters back into beam. Bottom: baffle to reduce noise and break coherence of scattered light. From Thorne and Blandford (2017).

### B. Gravitational noise

Humans working near a LIGO mirror create oscillating gravitational forces that might move the mirror more than does a gravitational wave. My wife, Carolee Winstein, is a bio-kinesiologist (expert on human motion). Using experimental data on human motion from her colleagues, we computed the size of this noise and concluded that, if humans are kept more than 10 meters from a LIGO mirror, the noise is acceptably small (Thorne and Winstein, 1999). This was used as a specification for the layout of the buildings that house the LIGO mirrors. Theory students scoped out noise produced by the gravitational forces of seismic waves in the Earth (Hughes and Thorne, 1998) and of airborne objects such as tumbleweeds (Creighton, 2008).

### C. Thermal noise

Thermal vibrations (vibrations caused by finite temperature) make LIGO's mirrors jiggle. These vibrations can arise in many different ways. Theory student Yuri Levin devised a new method to compute this thermal noise and to identify its many different origins (Levin, 1998). Most importantly he used his method to discover that thermal vibrations in the coatings of LIGO's mirrors (which previously had been overlooked) might be especially serious. This has turned out to be true: In the Advanced-LIGO interferometers, and likely in the next generation of gravitational interferometers, coating thermal noise is one of the two most serious noise sources; the other is quantum noise.

### D. Quantum noise and the standard quantum limit for a gravitational interferometer

*Quantum noise* is noise due to the randomness of the photon distribution in an interferometer's light beams. In each Initial LIGO interferometer (Parts I and II of this lecture), the

<sup>6</sup><https://www.black-holes.org/gw150914>.

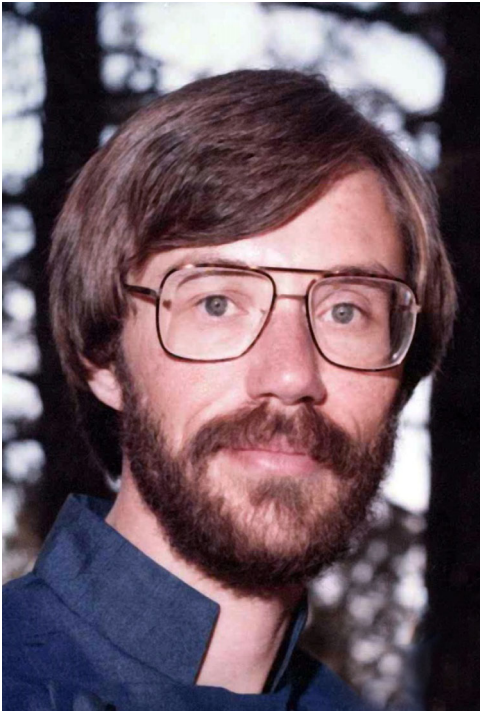


FIG. 16. Carlton M. Caves. Credit: Carlton M. Caves.

quantum noise had two parts: *photon shot noise*, caused by randomness in the arrival of photons at the photodetector (the interferometer's output); and *radiation-pressure noise*, caused by randomness in the bouncing of photons off the interferometers' mirrors, which makes the mirrors jiggle.

Both forms of quantum noise must arise from light-beam *differences* in the interferometers' two arms, since the interferometer output is sensitive only to differences.

In the late 1970s, there was much debate among gravitational-wave scientists over the physical origin of these differences. Theory postdoc Carlton Caves found the surprising answer (Caves, 1981): Both the radiation-pressure noise and the shot noise arise, he realized, from electromagnetic (*quantum electrodynamical*) vacuum fluctuations that enter the interferometer backward, from the direction of its output photodetector. These fluctuations beat against the laser light in the two arms to produce 1. radiation-pressure fluctuations (noise) that are opposite in the two arms, and 2. intensity fluctuations that also are opposite and that therefore exit from the interferometer into the output photodetector as shot noise; Figure 17.

With this new understanding in hand, Caves noted the rather obvious fact that, when one increases the laser intensity  $I$ , the shot noise goes down proportionally to  $1/\sqrt{I}$  and the radiation pressure goes up proportionally to  $\sqrt{I}$ ; so the quantum noise curve ( $h$  as a function of frequency  $f$ ) slides up and down a lower-limiting line as shown in Figure 18. That line is called the *standard quantum limit* (SQL) for an interferometer, and is given by Caves' simple formula

$$S_h^{1/2} = (8\hbar/mL^2\omega^2)^{1/2}. \quad (1)$$

Here  $S_h$  is the spectral density of the noise superposed on the gravitational-wave signal,  $\hbar$  is Planck's constant,  $m$  is the

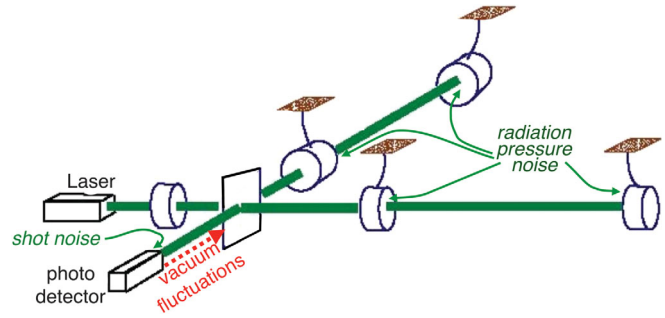


FIG. 17. Vacuum fluctuations entering the output port of a gravitational interferometer beat against laser light to produce shot noise in the output photodetector and radiation-pressure noise pounding on the mirrors.

mass of each of the interferometer's mirrors,  $L$  is the length of the interferometer's two arms and  $\omega$  is the gravitational wave's angular frequency.

In the late 1980s, Brian Meers at U. Glasgow (building on an idea of Ron Drever) proposed adding a signal recycling mirror to gravitational interferometers, in order to make them more versatile (see Weiss's and Barish's Parts I and II of this lecture), and by the late 1990s this new mirror was incorporated into the design for the future Advanced-LIGO interferometers. Strain and others used semiclassical (not fully quantum) theory to deduce the shot noise and radiation-pressure noise in these Advanced-LIGO interferometers. This was worrisome because Advanced LIGO was expected to operate very near its standard quantum limit, SQL, where the semiclassical analysis might be flawed. So theory postdoc Alessandra Buonanno and graduate student Yanbei Chen carried out a full quantum mechanical analysis of the noise.

Their analysis revealed surprises (Buonanno and Chen, 2001, 2003):

- The noise predictions of the semi-classical theory were wrong, so planning for Advanced LIGO would have to be modified, though not greatly.

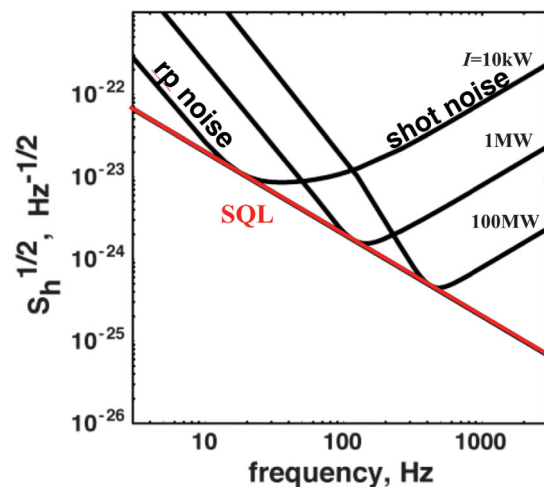


FIG. 18. The shot noise and radiation-pressure noise for various circulating powers  $I$  in the arms of the Initial LIGO interferometers.

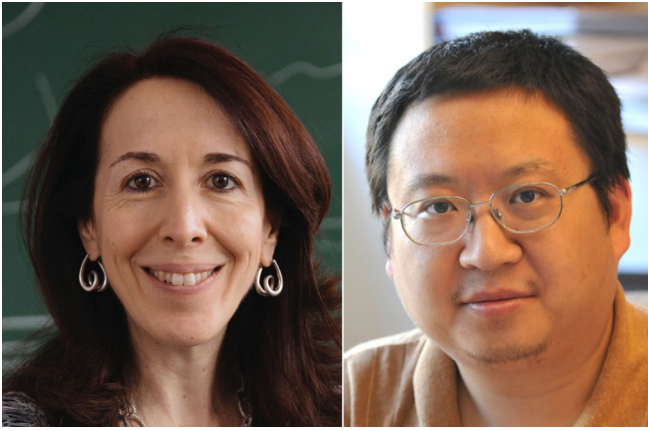


FIG. 19. Alessandra Buonanno and Yanbei Chen. Credits: Buonanno: S. Döring, Max Planck Society. Chen: Caltech.

- The interferometer’s signal recycling mirror triggers the beam’s light pressure in each arm to act as a frequency-dependent spring pushing against the mirrors, and so gives rise to an oscillatory, opto-mechanical behavior.
- The signal recycling mirror also creates quantum correlations between the shot noise and radiation-pressure noise. These correlations make it no longer viable to talk separately about shot noise and radiation-pressure noise; instead, one must focus on a single, unified quantum noise.
- These correlations also enable the Advanced-LIGO interferometer to beat Caves’ SQL by as much as a factor 2 over a bandwidth of order the gravitational-wave frequency.

#### E. Quantum fluctuations, quantum nondemolition, and squeezed vacuum

According to quantum theory everything fluctuates randomly, at least a little bit.

A half century ago, the Russian physicist Vladimir Braginsky argued (in effect) that in gravitational-wave detectors, when monitoring an object on which the waves act, one might have to measure motions so small that they could get hidden by quantum fluctuations of the object (Braginsky, 1968). Later, in the mid-1970s (Braginsky and Vorontsov, 1975), Braginsky realized that it should be possible to create *quantum nondemolition* (QND) technology to circumvent these quantum fluctuations.<sup>7</sup>

In 1980, Caves recognized that, although he derived his standard quantum limit [equation (1)] for an interferometer’s sensitivity by analyzing its interaction with light, this SQL actually has a deeper origin: it is associated with the quantum fluctuations of the centers of mass of the interferometer’s mirrors. The challenge, then, was to devise QND technology to circumvent those fluctuations and thereby beat their SQL.

Since the SQL is enforced by the electromagnetic vacuum fluctuations that enter the output port, Caves realized that a key

<sup>7</sup>For Braginsky’s own retrospective view of this work and subsequent developments up to 1996, see Braginsky and Khalili (1996).

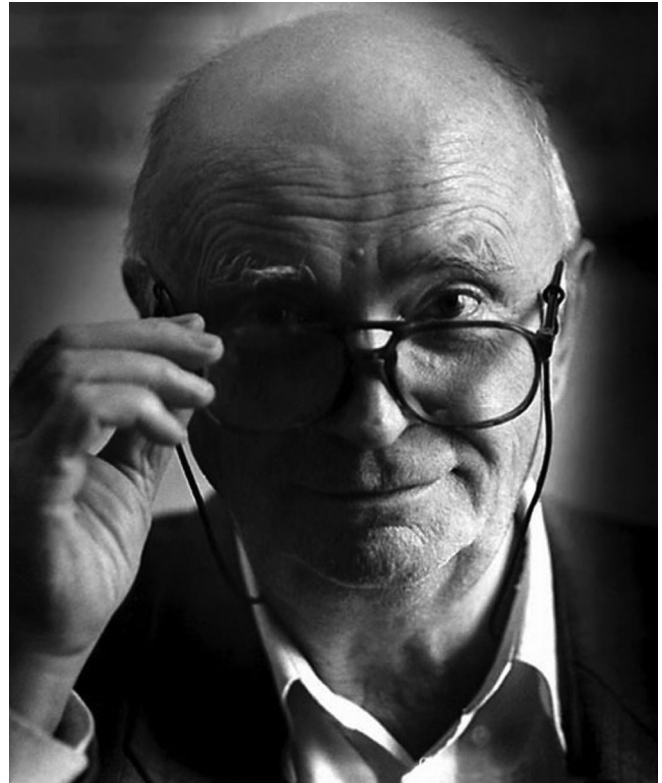


FIG. 20. Vladimir Braginsky. © Uspekhi Fizicheskikh Nauk 2012.

QND tool might be to modify those vacuum fluctuations—and thereby, through their radiation-pressure influence on the mirrors, modify the mirrors’ own quantum fluctuations.

More precisely, Caves (1981) proposed to reduce the electromagnetic vacuum fluctuations in one quadrature of each fluctuational frequency (e.g. the  $\cos \omega t$  quadrature) at the price of increasing the vacuum fluctuations in the other quadrature (e.g.  $\sin \omega t$ ). (The uncertainty principle dictates that the product of the fluctuation strengths for the two quadratures cannot be reduced, so if one is reduced, the other must increase.)

One quadrature is responsible for shot noise, and the other for radiation-pressure noise, Caves had shown; so by *squeezing the vacuum* in this way, one can reduce the shot noise at the price of increasing the radiation-pressure noise—which is the same thing as one achieves by increasing the laser light intensity. (This use of squeezed vacuum has since become very important: The original plan for bringing Advanced LIGO to its design sensitivity entailed pushing up to 800 kW the light power bouncing back and forth between mirrors in each interferometer arm. However, such high light power produces exceedingly unpleasant side effects; the mirrors have trouble handling it. Therefore, the new plan today, being implemented for LIGO’s next observing run in late 2018, entails injecting squeezed vacuum into the output port in precisely the manner Caves envisioned, instead of a corresponding increase in light power.)

In Advanced LIGO, shot noise dominates at high gravitational-wave frequencies (well above 200 Hz), radiation-pressure noise dominates at lower frequencies (well below

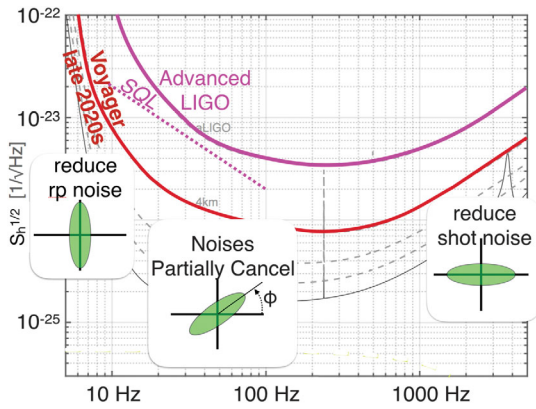


FIG. 21. Noise curves for Advanced LIGO at design sensitivity and the proposed Voyager interferometer, and the SQL. The green ellipses are the input squeezed vacuum at high, intermediate, and low frequencies, which enable Voyager to beat the SQL.

200 Hz). Therefore, it is advantageous to inject vacuum that is squeezed at a frequency-dependent quadrature  $\cos[\omega t - \varphi(\omega)]$ , which produces a shot-noise reduction ( $\varphi = 0$ ) at high frequencies, and a radiation-pressure reduction at low frequencies ( $\varphi = \pi/2$ ). At intermediate frequencies an amazing thing happens—as was discovered by Bill Unruh (Unruh, 1982) in 1981: the two noises, shot and radiation-pressure, partially cancel each other out! (See Figure 21.) As a result, the interferometer beats the SQL (it achieves quantum nondemolition), and with sufficient squeezing, it can do so by an arbitrarily large amount—in principle, but not in practice.

Although we have known this QND technique since 1983, in the 1980s and 1990s no practical method was known for producing the required frequency-dependent squeeze phase  $\varphi(\omega)$ .

In 1999, I discussed this problem in depth with my colleague Jeff Kimble (Caltech’s leading experimenter in squeezing and other quantum-information-related techniques), and he devised a solution: Squeeze the vacuum at a frequency-independent phase, then send the squeezed vacuum through one or two carefully tuned Fabry-Perot cavities (“optical filters”) before injecting it into the interferometer’s output port (Kimble *et al.*, 2002).

Among many different QND techniques that have been devised for LIGO interferometers [for a review see Danilishin and Khalili (2012)], this frequency-dependent squeezing, using Kimble filter cavities, is the one that currently looks most promising for future generations of gravitational interferometers: LIGO A+, Voyager, Cosmic Explorer, and Einstein Telescope (see Barish’s Part II of this lecture). A small amount of QND will be required in LIGO A+, and a substantial amount in all subsequent interferometers.

## VI. THE FUTURE: FOUR GRAVITATIONAL FREQUENCY BANDS

Electromagnetic astronomy was confined to optical and infrared frequencies until the late 1930s, when cosmic radio waves were discovered by Karl Jansky. Later, other frequency bands were enabled by telescopes flown above the earth’s

atmosphere: ultraviolet astronomy in the 1950s, and X-ray and gamma-ray astronomy in the 1960s. Over the decades since then, ever wider frequency bands have been opened up. It is common to speak of electromagnetic “windows” onto the universe, with each window being a frequency band in which astronomers work: the optical, infrared, radio, ultraviolet, X-ray and gamma-ray windows.

Gravitational waves are similar. Within the next two decades, we expect three more gravitational windows to be opened, so we will have the following:

- The *high-frequency gravitational window* (HF;  $\sim 10$  Hz to  $\sim 10,000$  Hz; wave periods  $\sim 100$  msec to  $\sim 0.1$  msec), in which LIGO, VIRGO and other ground-based interferometers operate.
- The *low-frequency gravitational window* (LF: periods minutes to hours) in which will operate constellations of drag-free spacecraft that track each other with laser beams, most notably the European Space Agency’s *LISA* (Laser Interferometer Space Antenna),<sup>8</sup> which is likely to be launched into space in 2030 or a bit later.
- The *very-low-frequency* gravitational window (VLF; periods of a few years to a few tens of years), in which *pulsar timing arrays* (PTAs),<sup>9</sup> are now operating and searching for gravitational waves.
- The *ultra-low-frequency* window (ULF: periods of hundreds of millions of years), in which primordial gravitational waves are predicted to have placed peculiar, observable polarization patterns onto the cosmic microwave radiation [Sec. 20.4 of Maggiore (2018)].

I will now describe LISA, PTAs, and CMB polarization in a bit more detail.

### A. LISA: The Laser Interferometer Space Antenna

LISA will consist of three spacecraft that track each other with laser beams. The spacecraft reside at the corners of an equilateral triangle with separations of a few million kilometers. This triangular constellation travels around the Sun in the same orbit as the Earth, following the Earth by roughly 20 degrees. Each spacecraft shields, from external influence, a *proof mass* (analog of a LIGO mirror), and uses thrusters to keep the spacecraft centered on the proof mass. The three proof masses, one in each spacecraft, move relative to each other in response to the tidal gravity of the Sun and the planets, and gravitational waves; and their relative motion is monitored by the laser beams using a technique called *heterodyne interferometry* (beating the incoming beam from a distant spacecraft against an outgoing beam). This is rather different from the type of interferometry used in LIGO.

The idea of a mission like LISA was discussed starting in 1974 by Peter Bender, Ronald Drever, Jim Faller, Rainer Weiss, and others. The presently planned orbital geometry (Fig. 22) was suggested by Faller and Bender in talks in 1981 and 1984 (Faller and Bender, 1984; Faller *et al.*, 1985). Bender then almost single handedly developed the LISA concept into a viable form through the 1980s and into the

<sup>8</sup><http://sci.esa.int/lisa/>

<sup>9</sup><http://www.ipta4gw.org>

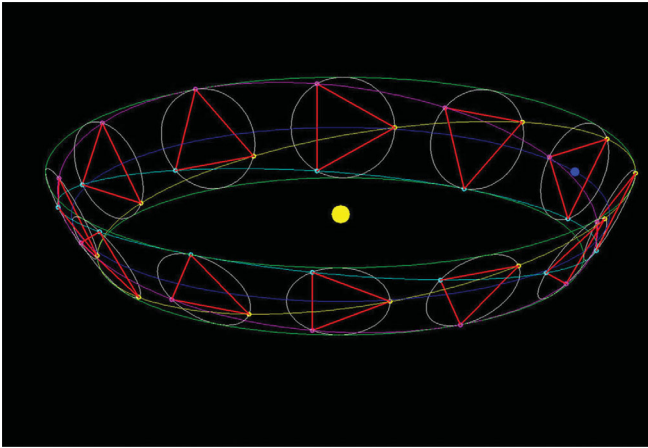


FIG. 22. The orbits of the three LISA spacecraft. Each follows a free-fall (geodesic) orbit around the sun, and their configuration remains nearly an equilateral triangle. Credit: HEPL, Stanford University.

1990s, leading NASA and ESA to develop a tentative plan for implementing it as a joint space mission. NASA dropped out in 2011 due mainly to cost overruns on the James Webb Space Telescope, leaving ESA to carry LISA studies forward alone, including a highly successful 2016 test of some of the most difficult technology, in the LISA Pathfinder Mission (Armano *et al.*, 2018). As of 2018 it appears that NASA may rejoin the LISA Mission as a junior partner to ESA and the launch might be as soon as 2030.

### B. PTAs: Pulsar timing arrays

A Pulsar Timing Array (PTA) consists of an array of several pulsars whose pulse periods are monitored with very high precision by one or more radio telescopes (Figure 24). Heuristically speaking, when a gravitational wave sweeps over the Earth, it causes clocks on Earth to speed up and slow down in an oscillatory pattern; so when compared with Earth clocks, all the pulsars appear to slow down and speed up synchronously.

A more accurate description of how a PTA works is this<sup>10</sup>: The gravitational wave creates an effective anisotropic index of refraction for the space through which the pulsars' radio waves travel. This index of refraction makes the pulsars appear to speed up and slow down synchronously by amounts that depend on the angles between the direction to the pulsar and the direction to the gravitational-wave source and the wave's polarization axes.

The idea of using pulsar timing to detect gravitational waves was conceived independently in the late 1970s by M. V. Sazhin and Steven Detweiler (Sazhin, 1978; Detweiler, 1979). Currently three radio-astronomy collaborations are attempting to detect gravitational waves using PTAs: the NANOGrav collaboration in North America, the European PTA, and the

<sup>10</sup>This is one way of describing the derivation of the response of a PTA to a gravitational wave [which, for example, is sketched all too briefly in Exercise 27.20 of Thorne and Blandford (2017)].



FIG. 23. Peter Bender (right) discussing the LISA mission concept with Ronald Drever (left) and Stan Whitcomb (middle) in Padova, Italy, in 1983. Credit: Peter Bender.

Parkes PTA (Australia); and the three also work in a loose worldwide collaboration called the International PTA.

The primary target of these collaborations is gravitational waves from gigantic black-hole binaries, weighing  $\sim 10^8$  to  $\sim 10^{10}$  suns. Current PTA sensitivities are adequate to detect these waves at the level of optimistic estimates, and success may well come in the next decade.

### C. CMB polarization

The cosmic microwave background (CMB) radiation, studied intensely by astronomers, last scattered off matter in the era when the primordial plasma was recombining to form neutral hydrogen (at universe age  $\sim 380,000$  years). In the 1990s, several theoretical astrophysicists (Seljak and Zaldarriaga,

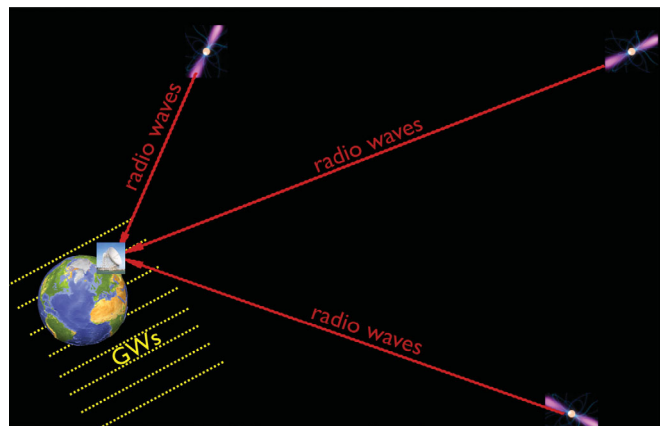


FIG. 24. Pulsar Timing Array: An array of three pulsars sends radio-wave pulses to Earth, whose observed timings are synchronously modulated by gravitational waves sweeping over the Earth.

1997; Kamionkowski, Kosowsky, and Stebbins, 1997) realized that primordial gravitational waves (waves from our universe's earliest moments), interacting with the recombining plasma, should have created a so-called *B-mode* pattern of polarization in the CMB. Searching for that pattern on the sky has become a "holy grail" for CMB astronomers, as it may reveal details of the primordial gravitational waves. The pattern has been found, but it can also be produced by microwave emission from dust particles and by synchrotron emission from electrons spiraling in interstellar magnetic fields. So the challenge now is to separate those two foreground contributions to the B-mode polarization from the gravitational-wave contribution [Sec. 20.4 of Maggiore (2018)]. It is plausible that this may be achieved in the coming decade.

## VII. THE FUTURE: PROBING THE UNIVERSE WITH GRAVITATIONAL WAVES

I conclude this lecture with some remarks about the science that is likely to be extracted from gravitational waves in the coming few decades. I shall discuss sources that include matter (multi-messenger astronomy), then the gravitational exploration of black holes, and finally observations of the first one second of the life of our universe. For details on all the sources I discuss, I recommend a book by Michele Maggiore (Maggiore, 2018).

### A. Multi-messenger astronomy

LIGO/Virgo's first binary neutron star (BNS), GW170817 (see Barish's Part II of this lecture) is a remarkable foretaste of the discoveries that will be made in the high-frequency band via multi-messenger astronomy. As ground-based interferometers improve:

- The event rate for BNSs will likely increase from approximately one per year now, to approximately one per month at LIGO design sensitivity (2020), to approximately one per day in Voyager (which could operate in the late 2020s; see Barish's Part II), to many per day in Cosmic Explorer and Einstein Telescope (which could operate in the 2030s; see Barish's Part II); and the richness and detail extracted from multi-messenger observations will increase correspondingly.
- We will almost certainly also watch many black holes tear apart their neutron-star companions in black-hole/neutron-star binaries, from which we might be able more cleanly to extract neutron-star physics via multi-messenger observations, than from BNSs.
- We will very likely also see multi-messenger emission from a variety of types of spinning, deformed neutron stars, including pulsars, magnetars, and perhaps low-mass X-ray binaries.
- If we are lucky, we will see gravitational waves from the births of neutron stars in supernovae, and through combined gravitational, neutrino, and electromagnetic observations, discover the mechanisms that trigger supernova outbursts.
- And if we are lucky, we will see electromagnetic emission from some merging black-hole binaries, due to the black holes' interaction with matter in their

vicinity, and we may thereby explore the black holes' near environments.

LISA and other low-frequency, space-based interferometers will participate in multi-messenger observations of a variety of astronomical objects and phenomena, including:

- White-dwarf binaries, and interactions between the two white-dwarf stars when they are very close together.
- AM CVn stars (a white dwarf that accretes matter from a low-mass helium-star companion).
- An enormous number of other binary star systems with gravitational-wave frequencies above about 0.1 mHz—with so very many between  $\sim 0.1$  mHz and  $\sim 2$  mHz that they will produce a stochastic background that dominates over LISA's instrumental noise.
- Possibly the implosion (collapse) of a few supermassive stars in galactic nuclei, to form supermassive black holes.

And of course, the most exciting prospect of all, is huge, unexpected surprises that entail multi-messenger emissions.

### B. Exploring black holes and geometrodynamics with gravitational waves

The high-, low-, and very-low-frequency bands cover BBH inspirals over the entire range of known black-hole masses, from a few solar masses to  $\sim 2 \times 10^{10}$  solar masses (Flanagan and Hughes, 1998).

In the high-frequency band of ground-based interferometers, BBHs with total mass up to about 1000 suns can be observed. As these interferometers improve, the rates of BBH events could increase from very roughly one per month in 2017 to a few per week at Advanced-LIGO design sensitivity ( $\sim 2020$ ), to as much as one per hour in Voyager (late 2020s), to every black-hole binary in the universe that emits in the high-frequency band, in Cosmic Explorer and Einstein Telescope (2030s). And with improving sensitivity, the maximum signal-to-noise ratio for BBH waves could increase from 24 today, to as much as 1000 in Cosmic Explorer and Einstein Telescope, with a corresponding increase in the accuracy with which the physics of black holes can be explored.

In the low-frequency band, LISA should see mergers of very massive black holes ( $\sim 10^3$  to  $\sim 10^8$  solar masses), with signal to noise as high as  $\sim 100,000$ , and corresponding exquisite accuracy for exploring geometrodynamics and testing general relativity.

LISA will likely also see many EMRIs: extreme mass-ratio inspirals, in which a small black hole or a neutron star or white dwarf travels around a very massive black hole on a complex orbit, gradually spiraling inward due to gravitational radiation reaction, and finally plunging into the massive hole. Figure 25 shows the spacetime geometry of the two black holes for the special case where the small hole is confined to the massive hole's equatorial plane; Figure 26 [from a simulation and movie by Drasco (2016)] shows a segment of a generic orbit for the small hole, when the large hole spins rapidly.

The complexity of the generic orbit results from the combined influence of the massive hole's very strong gravitational pull (very large *relativistic periastron shift*), the curvature of space around it (not depicted in the figure),



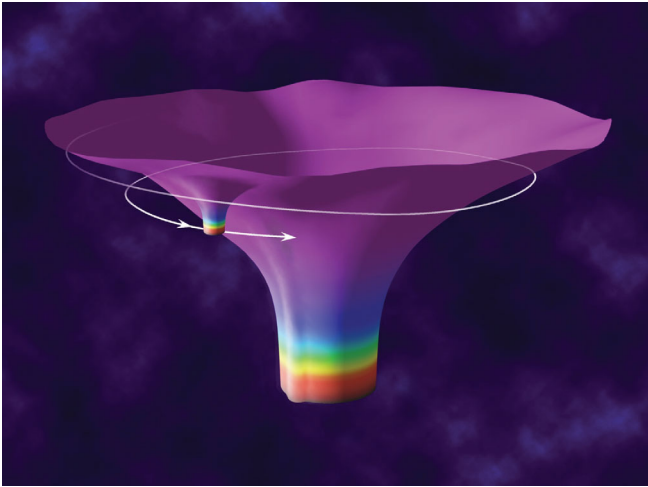


FIG. 25. Embedding diagram showing the spacetime geometry of a small black hole orbiting a large black hole, in the large hole's equatorial plane. Credit: NASA/JPL-Caltech.

and the whirling of space (dragging of inertial frames) caused by its spin. Over many months, the orbit explores a large portion of the space of the massive black hole, and so the complicated gravitational waveform it emits carries encoded in itself a highly accurate map of the massive hole's spacetime geometry (Ryan, 1995). A major goal of the LISA mission is to monitor the waves from such EMRIs, and extract the maps that they carry, thereby determining with high precision whether the massive hole's spacetime geometry is the one predicted by general relativity: the *Kerr geometry*.

The struggle to understand the quantum mechanical phenomenon of information loss into black holes has led to speculations that instead of a horizon down which things can fall, a black hole has a *firewall* (Almheiri *et al.*, 2013); and also speculations that the firewall modifies the spacetime geometry from that of Kerr outside but near the firewall's

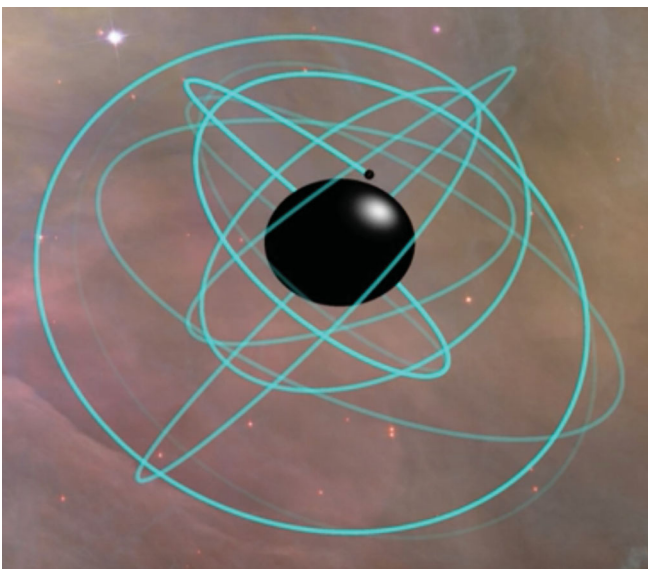


FIG. 26. Segment of generic orbit for a small black hole orbiting a rapidly spinning large black hole. Credit: Steve Drasco.

location [see, e.g., Giddings (2016)]. LISA's mapping project will search for any such modification. By this mapping project, LISA can also search for unexpected types of massive, compact objects, whose spacetime geometries differ from that of Kerr, for example, naked singularities that are being orbited by much smaller bodies.

### C. Exploring the first one second of our Universe's life

Every known type of particle or radiation, except gravitational waves, is predicted to be trapped by the universe's hot, dense plasma during the first one second of our universe's life. Therefore, gravitational waves are our only hope for directly observing what happened during that first one second.

Among the predictions that such observations might test is the origin of the electromagnetic force—one of the four fundamental forces of Nature. Theory predicts that, when the universe was very young and very hot, the electromagnetic force did not exist. In its place there was an *electroweak force*. As the universe expanded and cooled through an age of  $\sim 10^{-11}$  seconds and a temperature of  $\sim 10^{15}$  K, there was, according to theory, a *phase transition* in which the electroweak force came apart, giving rise to two new forces: the electromagnetic force, and the weak nuclear force.

If this was a so-called *first-order* phase transition (which it may well not have been), then it is predicted to be like the transition from water vapor to liquid water when the vapor is cooled through 100 °C: the transition should have occurred in bubbles analogous to water droplets. Inside each bubble, the electromagnetic force existed; outside the bubbles, it did not exist. Theory predicts that these bubbles expanded at very high speeds, collided, and produced, in their collisions, stochastic gravitational waves. As the universe expanded, the wavelengths of these waves also expanded, until today, 13.8 billion years later, the wavelengths are expected to be in LISA's frequency band [see, e.g., Sec. 22.4 of Maggiore (2018)]. One of LISA's goals is to search for these stochastic gravitational waves produced by the birth of the electromagnetic force.

LIGO could see gravitational waves produced by a similar first-order phase transition when the universe was far younger,  $\sim 10^{-22}$  seconds, and far hotter,  $\sim 10^{21}$  K. In logarithmic terms, this time and temperature are roughly halfway between the electroweak phase transition and the phase transition associated with grand unification of the fundamental forces. Unfortunately, this is an epoch at which no phase transition is predicted by our current understanding of the laws of physics.

Gravitational waves are so penetrating—so immune to absorption or scattering by matter—that they could have been generated in our universe's big-bang birth, and traveled to Earth today unscathed by matter, bringing us a picture of the big bang.

This picture, however, is predicted to have been distorted by *inflation*, the exponentially fast expansion of the universe that is thought (with some confidence) to have occurred between age  $\sim 10^{-36}$  seconds and  $\sim 10^{-33}$  seconds. More specifically, inflation should have *parametrically amplified* whatever gravitational waves came off the big bang. This amplification may well have made the primordial gravitational waves strong enough for detection, but the amplification will also have

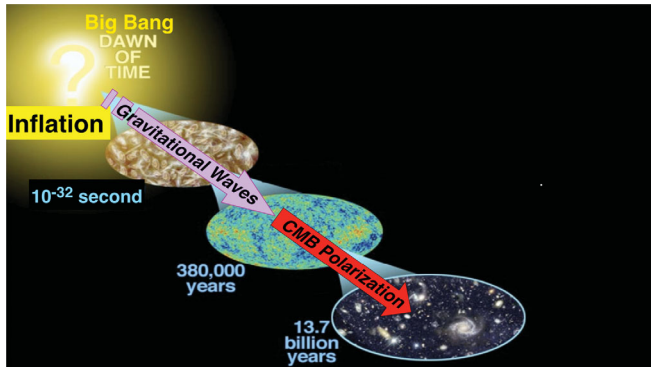


FIG. 27. Primordial gravitational waves, amplified by inflation at universe age  $\sim 10^{-36}$  to  $\sim 10^{-33}$  sec, interact with primordial plasma at age 380,000 years, placing a polarization imprint on the CMB which is observed today at age 13.8 billion years. Credit: Adapted from WMAP # 020622/NASA/WMAP Science Team.

distorted the waves, so that the spectrum humans see is a *convolution* (combination) of what came off the big bang, and the influence of inflation.

Remarkably, we have the possibility, by the middle of this (twenty first) century, to observe these primordial gravitational waves in two different frequency bands:

- In the extremely low-frequency band, by the B-mode polarization pattern that the waves place on the cosmic microwave background radiation, CMB; see above and Figure 27.
- At periods of seconds, between the high-frequency band and the low-frequency band, using a proposed successor to LISA: the Big Bang Observer (Phinney *et al.*, 2004), which consists of several constellations of light-beam-linked spacecraft in interplanetary space (Figure 28).

Theorists' conventional wisdom dictates that what came off the big bang was the weakest gravitational waves allowed by the laws of Nature: vacuum fluctuations of the gravitational field. Inflation's parametric amplification was so strong that even beginning with just vacuum fluctuations, the resulting primordial gravitational waves are likely to be strong enough for observation by both of these detectors, in both frequency bands—bands that differ in frequency and in wave period and wavelength by a factor of  $\sim 10^{15}$ .

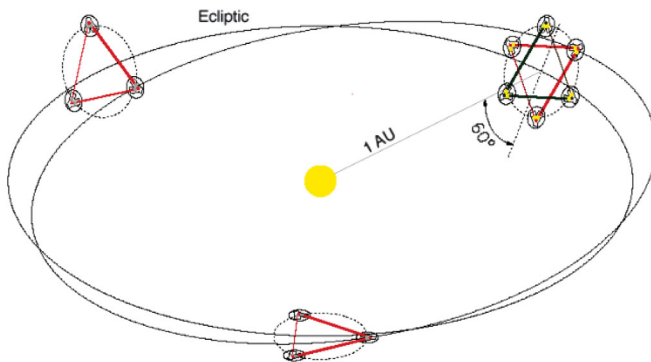


FIG. 28. The Big Bang Observer's constellations of spacecraft, in the same orbit around the Sun as the Earth. Credit: Sterl Phinney.

I am skeptical of theoretical physicists' conventional wisdom, as I have seen it fail spectacularly in several ways during my career. I look forward to the possibility, indeed the likelihood, that the observations will differ from this conventional wisdom in one or both frequency bands, and that the observations will reveal enough about the birth of the universe to give crucial guidance to physicists who are trying to discover the laws of quantum gravity: the laws that governed the universe's big-bang birth.

## VIII. CONCLUSION

Four hundred years ago, Galileo built a small optical telescope and, pointing it at Jupiter, discovered Jupiter's four largest moons; and pointing it at our moon, discovered the moon's craters. This was the birth of electromagnetic astronomy.

Two years ago, LIGO scientists turned on their Advanced-LIGO detector and, with the data-analysis help of VIRGO scientists, discovered the gravitational waves from two colliding black holes 1.3 billion light years from Earth.

When we contemplate the enormous revolution in our understanding of the universe that has come from electromagnetic astronomy over the four centuries since Galileo, we are led to wonder what revolution will come from gravitational astronomy, and from its multi-messenger partnerships, over the coming four centuries.

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